MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2022/2023 SESSION (TERM ID 2220)

PPS0016 – INTRODUCTION TO PROBABILITY AND STATISTICS

(All sections / Groups)

12 JULY 2023
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of 6 pages (excluding the cover page) with 5 questions.
2. Answer all questions.
3. Unless stated otherwise, if an answer is given as a decimal, it should be rounded to four significant figures.
4. Write your answers in the Answer Booklet provided.
5. Show all relevant steps to obtain maximum marks.

Question 1 [20 marks]

The accompanying data are the drive-through service times (in seconds) for a fast food restaurant. Times begin when a vehicle stops at the order window and end when the vehicle leaves the pickup window.

116	131	147	120	126	166	77	136	133	121
182	146	138	110	174	110	188	120	105	79
120	188	159	83	102	78	224	199	174	137
142	161	195	253	231	259	100	128	84	153
151	145	171	160	124	157	136	148	145	166

(a) Determine the class size by using the range of the data and if given the number of class is 8. [2 marks]

(b) Complete the following table with 8 classes. Use the smallest value as the lower limit of the first class. [10 marks]

Class Limit	Frequency	Midpoint	Midpoint × Frequency	Midpoint^2 × Frequency
			⋮	
Total				

(c) By using the value found in table (b), find the following:

(i) the mean of the service time. [2 marks]

(ii) the sample standard deviation of the service time. [2 marks]

(iii) the value of k if half of the data have service time less than k. [2 marks]

(d) State the variable in this problem. Is it discrete or continuous? [2 marks]

Continued...

Question 2 [20 marks]

(a) In a survey of 40 people on the film genres that they like, the results are as follows:

18 like horror movies

19 like action movies

21 like science fiction movies

9 like both horror and action movies

11 like both science fiction and action movies

5 like all the three genres

4 do not watch the three genres

(i) Draw a Venn diagram to visualize the above information. Use H, A, and S as the events that a person likes horror, action, and science fiction movies respectively. Then, find the number of people who likes only horror and science fiction movies. [5 marks]

(ii) Find the probability that a randomly selected person

(1) likes only one genre [2 marks]

(2) like at least two genres, [2 marks]

(iii) Write the event that a person likes only horror and action movies using H, A, S, $\cap$, $\cup$, and complement ', whichever necessary. [1 mark]

(b) Two chess players, Hello and Kitty, are playing each other in a series of games. The probability that Hello wins the first game is 0.3 If Hello wins any game, the probability that she wins the next is 0.4. Otherwise, the probability that Hello wins the next game if Hello loses the first game is 0.2. Both will play a series of two games.

(i) Draw a tree diagram to represent the given information. No need to show the leaves. [6 marks]

(ii) Find the probability that Hello wins a game but loses in the other. [2 marks]

(iii) Given that Hello loses the second game, what is the probability that Hello wins the first one? [2 marks]

Continued...

Question 3 [20 marks]

(a) Bag A contains three discs with the numbers 1, 2, and 3 on them. Bag B contains four discs with the numbers 1, 2, 3, and 3 on them. A disc is drawn at random from bag A and the number is noted. A second disc is then drawn from bag B and the number is noted. The discrete random variable X is the sum of the two numbers.

(i) Construct the probability mass distribution for X. [5 marks]

(ii) Illustrate the distribution of X and state the shape of the distribution. [3 marks]

(iii) If the experiment is repeated a lot of times, find the mean of the sum. [2 marks]

(b) Given the function

$$f(x)=\begin{cases}\frac{1}{7}x^2-1, & 3\le x\le 4\\ 0, & \text{otherwise}\end{cases}.$$

(i) Sketch $f(x)$. [2 marks]

(ii) Determine whether $f(x)$ is a probability density function. Show appropriate evidence to support your claim. [4 marks]

(c) A random variable X has probability density function given by:

$$f(x)=\begin{cases}\frac{1}{6}, & -2\le x<0\\ kx^3, & 0\le x\le 4\\ 0, & \text{otherwise}\end{cases}$$

Show that the value of k is $\frac{1}{96}$. [4 marks]

Continued...

Question 4 [20 marks]

(a) Vaccine A protects 85% of the people who take it from Pneumococcal disease. A random sample of 50 people who take the vaccine is drawn, and the number of people who are protected from Pneumococcal disease, X, is recorded. Analyze whether the above experiment is a binomial experiment or not by determining whether it satisfies each of the conditions of a binomial experiment. [5 marks]

(b) On average, the number of bent rods produced by a machine is 4.8 in 400 rods. The number of bent rods produced follows a Poisson distribution.

(i) What is the probability that there are 2 bend rods in a random sample of 400 rods? [2 marks]

(ii) What is the probability that there are 4 bend rods in a random sample of 800 rods? [2 marks]

(c) 8% of the students in a class fail mathematics.

(i) Ali, Ahmad, and Chandran are good friends. What is the probability that they all fail mathematics? [2 marks]

(ii) The class has 40 students. What is the expected number of students who pass mathematics? [2 marks]

(iii) Name the probability distribution in this problem. [1 mark]

(d) Z is a standard normal random variable. Find the value of k in the following by using the cumulative standardized normal distribution table provided in the appendix.

(i) $P(Z > k) = 0.2$ [3 marks]

(ii) $P(1.43 < Z < k) = 0.34$ [3 marks]

Continued...

Question 5 [20 marks]

(a) Given the population data 3, 5, 5, 5, and 6.

(i) Construct the sampling distribution of the sample mean for samples of size 3 by filling in the following two tables. Use subscript to differentiate the same data values, for example, 5_1, 5_2 and 5_3. [9 marks]

No	Sample	Sample mean

⋮

Sample mean	Frequency	Probability of sample mean

⋮

(ii) Calculate the mean of the sampling distribution of sample mean from the table built in (a)(i). [2 marks]

(iii) Find the probability that a sample mean is greater or equals to the mean of the sampling distribution of sample mean found in (a)(ii). [2 marks]

(b) The volume of liquid per bottle of a particular drink is normally distributed with a mean and standard deviation of $\mu = 500$ mℓ and $\sigma = 5$ mℓ, respectively.

(i) Suppose Ali takes a random sample of 50 bottles, find the probability that the mean volume of this sample, $\bar{x}$, is 495 mℓ or more. [6 marks]

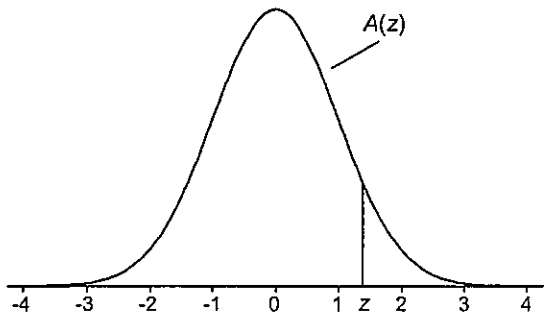
(ii) Do you need to use the central limit theorem in (b)(i)? [1 mark]

Continued...

Appendix

TABLE A.1

Cumulative Standardized Normal Distribution



$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:

z	$A(z)$	
1.645	0.9500	Lower limit of right 5% tail
1.960	0.9750	Lower limit of right 2.5% tail
2.326	0.9900	Lower limit of right 1% tail
2.576	0.9950	Lower limit of right 0.5% tail
3.090	0.9990	Lower limit of right 0.1% tail
3.291	0.9995	Lower limit of right 0.05% tail

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999							

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